Listening to the Doppler shift of visible light

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Doppler shift of light can be reproduced audibly with minimal apparatus: a commercial He–Ne laser, two mirrors, a photodetector, and a stereo amplifier with speakers. A dual-trace oscilloscope and signal generator can be added to obtain a numerical value for the laser wavelength. The method can be extended to display optical spectra directly on an audio spectrum analyzer.

INTRODUCTION

The Doppler shift in optical frequency of a moving source is a phenomenon well known to students of relativity and astrophysics. However, actual measurement of this effect usually involves a rather sophisticated apparatus not found in the undergraduate physics laboratory. We describe here a simple method for demonstration, and optional measurement, of this shift for equivalent source velocities on the order of 1 cm/sec. A feature of the method is that audible output is generated, so that the listener perceives an effect reminiscent of the gliding whistle of a passing train.

The technique was inspired by the interesting observation of Brickner et al.,\(^1\) that the speed of light can be measured fairly accurately with a uhf television receiver, a photodiode, and a He–Ne laser. In their arrangement, the equipment list is modest chiefly because internal laser beat frequency can be read directly from the uhf channel selector. In our approach we exploit instead the high gain of a typical high-fidelity amplifier and the innate sensitivity of the human ear. A good demonstration of the Doppler shift of light is thus achieved with the following equipment: a He–Ne laser,\(^3\) a phototransistor,\(^4\) a stereo amplifier with speakers, and two mirrors. If numerical results are desired, an audio signal generator and dual-trace oscilloscope complete the list. The main numerical result is a value for the laser wavelength. We shall also indicate how an audio spectrum analyzer might be used to display optical spectra of polychromatic sources.

THEORY

For an optical source of frequency \(f\) moving away from the observer with speed \(v\), the observed frequency \(f'\) is given by the approximate (nonrelativistic) Doppler formula:\(^3\)

\[
\frac{f}{f'} = 1 - \frac{v}{c},
\]

(1)

where \(c\) is the speed of light. In the present treatment, \(v\) will be of the order of 1 cm/sec, so that (1) is valid to high precision. If the moving source is actually an irradiated mirror, there is an extra factor of two due to doubled apparent recession of the radiation source. Thus for mirror speed \(v\), the fractional Doppler shift is given by

\[
\frac{\Delta f}{f} = \frac{f' - f}{f} = -2\frac{v}{c}.
\]

(2)

If the source beam and its moving-mirror reflection are brought together onto a detector which is sensitive to intensity, that is a squared wave amplitude, there will be a detector output component that oscillates at the beat frequency \(\Delta f = f' - f\). The existence of this Doppler beat signal can be understood from various points of view, if it is kept in mind that \(v/c\) is small and therefore classical wave theory applies.

One point of view is that the detector acts as a simple mixer. At any point on the detecting surface, the wave amplitude is the sum of a carrier signal (frequency \(f\)) and a single sideband (frequency \(f'\)). The intensity at this point will generate a cross-term current of frequency \(\Delta f\) which, when integrated over the detecting surface, will yield, in general, a nonzero Doppler beat current suitable for amplification and eventual audio reproduction.

A second equivalent explanation of the Doppler beat current starts with the observation that the apparatus is a simple interferometer. As the mirror moves away from the detector, the overall source-mirror-detector pathlength elongates, so that the interference pattern on the detecting surface changes in time. For example, as the mirror moves through one quarter of one wavelength, the interference at a particular detector point moves through one extremum. It follows that intensity fluctuations of frequency \(|2\nu/\lambda| = \Delta f\) occur at such a detector point, and again we integrate over the surface to get a Doppler beat current.

From the standpoint of signal theory, the equivalence of the above descriptions of the Doppler beat can be traced to the fact that fixed frequency shift is equivalent to a phase shift that advances linearly in time.

It is of interest that the Doppler beat frequency is just twice the number of wavelengths traversed per unit time by the mirror. Thus it is not surprising that the mirror speed must be small if the amplified beat current is to be in the audio frequency range.

APPARATUS

The Doppler beat photocurrent develops a voltage across a 30-\(\Omega\) impedance (see Fig. 1). This voltage is fed to input \(A\) of a stereo amplifier, with a speaker connected to the corresponding output \(A\). The other channel of the amplifier is dedicated to the purpose of moving the mirror. Thus a second speaker, to which a mirror is attached, is connected to output \(B\), while input \(B\) is optionally available for low-frequency signal input. Generally, the instantaneous Doppler shift will be proportional to the derivative of the signal at input \(B\).

It was found that very little reflected moving-mirror signal is enough for audible effect. In fact, a hemispherical metallic dust cap on the moving speaker worked reasonably well, even though the return beam in this case was highly divergent.

Fig. 1. Diagram of experimental Doppler shift apparatus. Laser light from both fixed and moving mirrors is mixed onto a photodetector. Resulting Doppler beats for sufficiently low speaker-mirror speeds are audible and measurable. The optional signal input can be used to drive the mirror in precise fashion in order to measure laser wavelength.

The apparatus as described produces Doppler whistles as the speaker mirror is gently perturbed. Even room vibrations and air currents will cause perceptible whistling. The optional signal input $B$ can be touched by hand to generate a small 60-Hz hum signal for the moving speaker. The dependence of Doppler frequency on mirror speed can be demonstrated this way, in the sense that many frequencies can be discerned as the speaker mirror so oscillates.

More graphic demonstration can be achieved by substituting a signal generator, set at a few Hz, at input $B$. Then null output on channel $A$, which occurs for extremal mirror excursions, can be observed.

**WAVELENGTH MEASUREMENT**

The apparatus in Fig. 1, together with a dual-trace oscilloscope and a signal generator, allows a reasonable measurement of the wavelength of laser light. The experimental procedure runs as follows:

(a) Fix the generator frequency to be $f_0 = 10$ Hz. Assume that the mirror displacement is given by

$$ x(t) = K V_g \sin(2\pi f_0 t), $$

for an appropriate constant $K$, where $V_g$ is the peak generator voltage. Measure $K$ directly for a large value of $V_g$.

(b) Connect one trace of the oscilloscope to the audible Doppler output and the other trace to the terminals of the mirror speaker. Reduce $V_g$ until the maximum Doppler shift [which appears on the oscilloscope screen at those times for which $x(t)$ is zero] is $\Delta f \approx 1000$ Hz as judged from the sweep setting.

(c) By differentiating the formula for $x(t)$ and using (2) we infer that the maximum Doppler shift is

$$ \Delta f = 2\pi f_0 K V_g \left(2f_0/c\right), $$

from which the wavelength is determined as

$$ \lambda = c/\Delta f = 4\pi f_0 K V_g / \Delta f. $$

Since all quantities on the right-hand side are measured, a value for $\lambda$ is obtained.

Using this procedure we found the laser wavelength to be $6400 \pm 500$ Å. This is in agreement with the established value of 6328 Å. The main error in our procedure arises from direct measurement of $K$ in mm/V.

**EXTENSION OF THE METHOD**

It is of interest that for fixed mirror drive signal, the maximum Doppler shift is proportional to the frequency $f$ of the light source. This means that in principle different optical components from a composite source are distinguishable. We sketch here a method for a real-time display of optical spectra based on this observation.

First, the mirror speaker is to be driven with a linear ramp signal. For best results, about a 0.2-Hz repetition rate is appropriate, with a total excursion of about 0.5 cm for the speaker cone. Second, a spectrum analyzer is to be swept with the ramp itself as trigger signal. The analyzer is to be adjusted so that the 0–50-kHz band is displayed only during the slow rise of the ramp signal, with signal input taken from the audible output of the amplifier. The resulting spectral display will be a replica of the true optical spectrum.

We observed with this technique that a strong white-light source, masked by a pinhole, indeed showed wideband spectral components. The He-Ne laser itself shows a well-defined spike corresponding to 6328 Å. It was found to be quite difficult to obtain precise quantitative spectral measurements with this technique. The difficulty was traced to the natural instabilities and extraneous mechanical modes of the speaker cone. Still, the method is worth pursuing, especially if an optically precise moving-mirror carrier can be devised.

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3 Spectra Physics Model 145-02, 2-mW He-Ne laser.
4 Motorola MRD3052 NPN phototransistor. In Ref. 1 a fast photodiode was used. Such a fast device can be used here, in which case 15 V and the resistor can be eliminated, but lower sensitivity can be expected.
5 Tektronix 5103N mainframe with spectrum analyzer 514N.